The moment you have been waiting for . . . .

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

[[but perhaps only for certain xs]]

Interval ([radius]) of convergence and where is that interval centered?

$$
\begin{array}{r|l}
 & 1 + x + x^2 + \cdots \\
\hline
1-x & 1 \\
 & -(1-x) \\
\hline
 & x \\
 & -(x-x^2) \\
\hline
 & x^2 \\
 & -(x^2-x^3) \\
\hline
 & x^3 \\
 & \vdots
\end{array}
$$

Mar 27-7:45 AM

Know $\frac{1}{1-x} \approx 1+x+x^2+x^3+\cdots$

Can I figure out what $\frac{1}{1+x} = ?$

$$\frac{1}{1+x} = \frac{1}{1-(-x)}$$

$$= 1+(-x)+(-x)^2+(-x)^3+(-x)^4+\cdots$$

$$= 1-x+x^2-x^3+x^4 \cdots$$

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$$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)}$$

$$= 1 + (-x^2) + (-x^2)^2 + (-x^2)^3 + \cdots$$

$$= 1 - x^2 + x^4 - x^6 + x^8 - \cdots$$

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$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

$$\int \frac{1}{1-x}\,dx = -\ln|1-x|$$

$u = 1-x$

$du = -dx$

$$= x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots + C$$

$$-\ln\left|1-x\right|\Big|_{x=0} = 0$$

$$\therefore C = 0$$

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Creating an infinite series....a second way

$$\sin x = \sum_{k=0}^{\infty} a_k x^k = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$$

$$0 = \sin(0) = a_0 + 0 + 0 + \dots = a_0$$

$$\cos x = a_1 + 2a_2 x + 3a_3 x^2 + \dots$$

$$1 = \cos(0) = a_1 + 0 = a_1$$

$$-\sin x = 2a_2 + 6a_3 x + 12a_4 x^2$$

$$0 = -\sin 0 = 2a_2$$

$$-\cos x = 6a_3 + 24a_4 x + \dots$$

$$-1 = 6a_3$$

$$a_3 = \frac{-1}{6} = \frac{-1}{3\cdot 2}$$

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \dots$$

Mar 27-8:19 AM

A Maclaurin series is a "derivative-generated" series "centered on" $x=0$

$f^{(k)}$ is the k^{th} derivative

$0! = 1$

$$\sum_{k=0}^{\infty} \frac{f^{(k)}}{k!} x^k$$

Mar 27-7:45 AM

HW find Maclaurin series for $\cos x$ and e^x

& write them in sigma notation

Mar 27-8:18 AM

The moment you have all been waiting for :)

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

$$\begin{array}{r|l} & 1 + x + x^2 + x^3 + \cdots \\ \hline 1-x & 1 \\ & -(1-x) \\ \hline & x \\ & -(x - x^2) \\ \hline & x^2 \\ & \vdots \end{array}$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

"converges" for x s around $x=0$

when does an inf series converge

[to the f^n I'm interested in]

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

$$\frac{1}{1+x} = \frac{1}{1-(-x)}$$

$$= 1 + (-x) + (-x)^2 + (-x)^3 + (-x)^4 + \cdots$$

$$= 1 - x + x^2 - x^3 + x^4 - x^5 + \cdots$$

$$\frac{1}{1+x^2} = 1 + (-x^2) + (-x^2)^2 + (-x^2)^3 + (-x^2)^4 + \cdots$$

$$= 1 - x^2 + x^4 - x^6 + x^8 - \cdots$$

Mar 27-11:55 AM

$$\int \frac{1}{1-x}\,dx = -\ln|1-x| + c \ldots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots$$

$$-\ln|1-x| \cancel{+c} = C + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots$$

when $x = 0$

$$-\ln|1-x| = -\ln|1-0| = 0$$

$$\therefore C = 0$$

$$-\ln|1-x| = x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots$$

what about sin x ?

$\sin x = \cancel{a_0} + a_1 x + \cancel{a_2 x^2} + a_3 x^3 + \cancel{\cdots}$

let $x=0$

$0 = \sin 0 = a_0$

$\cos x = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \cdots$

$1 = \cos 0 = a_1$

$-\sin x = 2a_2 + 6a_3 x + 12a_4 x^2 + 20a_5 x^3 \cdots$

$0 = -\sin 0 = 2a_2$ so $a_2 = 0$

$-\cos x = 6a_3 + \cdots$

$-1 = 6a_3 \Rightarrow a_3 = \frac{-1}{6} \cdots$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots$$

Mar 27-12:08 PM

A Maclaurin series is a derivative-generated series whose interval of convergence is centered at $x=0$

$$\text{series} = \sum_{k=0}^{\infty} \frac{f^{(k)}}{k!} x^k$$

$f^{(k)} =$ the k^{th} derivative

- the 0^{th} derivative is the original f^n
- $0! = 1$

$$\sin x = \sum_{K=0}^{\infty} \frac{f^{(2K+1)}(0)}{(2K+1)!} x^{(2K+1)} = \sum_{K=0}^{\infty} \frac{(-1)^K}{(2K+1)!} x^{2K+1}$$

HW: find Maclaurin series for $\cos x$ and e^x & write in sigma notation

Mar 27-12:15 PM